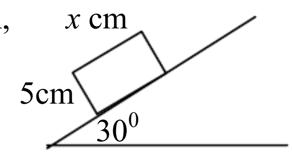
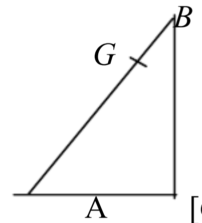
MECHANICS (C) UNIT 2 TEST PAPER 3

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A solid rectangular block, whose cross^{-1}section measures x cm by 5 cm, is placed gently on a rough plane inclined at 30^0 to the horizontal, as shown. The coefficient of friction between the block and the plane is 0·6. Show that the block does not slide down the plane and find the smallest value of x for which the block will not topple. [4]

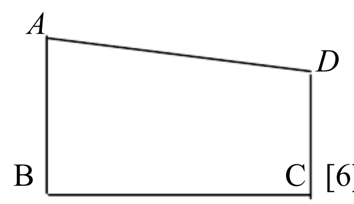


2. A non^{-1}uniform ladder AB, of length $3a$, has its centre of mass at G, where $AG = 2a$. The ladder rests in limiting equilibrium with the end B against a smooth vertical wall and the end A resting on rough horizontal ground. The angle between AB and the horizontal in this position is α, where $\tan \alpha = {}^{14}/_{9}$. Calculate the coefficient of friction between the ladder and the ground. [6]



3. A lorry of mass 4200 kg can develop a maximum power of 84 kW. On any road the lorry experiences a non^{-1}gravitational resisting force which is directly proportional to its speed. When the lorry is travelling at 20 ms^{-1} the resisting force has magnitude 2400 N. Find the maximum speed of the lorry when it is
   (i) travelling on a horizontal road, [3]
   (ii) climbing a hill inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{7}$. [5]

4. A uniform lamina is in the form of a trapezium $ABCD$, as shown. AB and DC are perpendicular to BC. $AB = 17$ cm, $BC = 21$ cm and $CD = 8$ cm.
   (i) Find the distances of the centre of mass of the lamina from
   (a) AB, (b) BC. [6]



   The lamina is freely suspended from C and rests in equilibrium.
   (ii) Find the angle between CD and the vertical. [2]

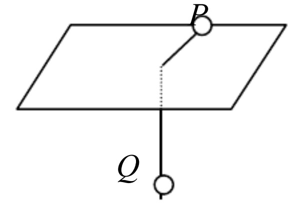
5. Two railway trucks, P and Q, of equal mass, are moving towards each other with speeds $4u$ and $5u$ respectively along a straight stretch of rail which may be modelled as being smooth.They collide and move apart. The coefficient of restitution between P and Q is e.
   (i) Find, in terms of u and e, the speed of Q after the collision. [5]
   (ii) Show that $e > \frac{1}{9}$. [2]
   Q now hits a fixed buffer and rebounds along the track. P continues to move with the speed that it had immediately after it collided with Q.
   (iii) Prove that it is impossible for a further collision between P and Q to occur. [3]

6. A stone, of mass 1·5 kg, is projected **horizontally** with speed 4 ms^{-1} from a height of 7 m above horizontal ground.
   (i) Show that the stone travels about 4·78 m horizontally before it hits the ground. [3]
   (ii) Find the height of the stone above the ground when it has travelled half of this horizontal distance. [3]
   (iii) Calculate the potential energy lost by the stone as it moves from its point of projection to the ground. [2]
   (iv) Showing your method clearly, use the principle of conservation of energy to find the speed with which the stone hits the ground. [2]

(v) State **two** modelling assumptions that you have made in answering this question. [2]

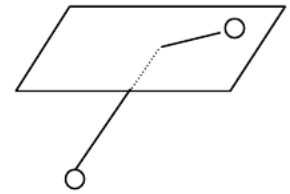
7. Two identical particles P and Q are connected by a light inextensible string passing through a small smooth edged hole in a smooth table, as shown.

P moves on the table in a horizontal circle of radius 0·2 m and Q hangs at rest.



(i) Calculate the number of revolutions made per minute by P. [4]

Q is now also made to move in a horizontal circle of radius 0·2 m below the table. The part of the string between Q and the table makes an angle of 45^0 with the vertical.



(ii) Show that the numbers of revolutions per minute made by P and Q respectively are in the ratio $2^{\frac{1}{4}} : 1$. [8]

MECHANICS 2 (C) TEST PAPER 3 : ANSWERS AND MARK SCHEME

1. $\tan 30^0 < 0{\cdot}6$, so block does not slide — B1
   Topples if $^x/_5 < \tan 30^0$, so does not topple if $x \geq 2{\cdot}89$ — M1 A1 A1 — 4
2. Let reactions be R at ground, S at wall
   $M(A) : W(2a \cos \alpha) = S(3a \sin \alpha)$ $S = 2W \div 3 \tan \alpha = {}^3/_7\, W$ — M1 A1 A1
   Resolve : $R = W$, $\mu R = S$ $\mu = S \div W = {}^3/_7$ — B1 M1 A1 — 6
3. (i) $2400 = 20k$ $k = 120$ $84000 = v(120v)$ $v = 26{\cdot}5 \text{ ms}^{-1}$ — M1 A1 A1
   (ii) $P = v(600g + 120v)$ $120v^2 + 5880v - 84000 = 0$ — M1 A1 A1
   $v^2 + 49v - 700 = 0$ $v = (-49 + \sqrt{5201})/2 = 11{\cdot}6 \text{ ms}^{-1}$ — M1 A1 8
4. (i) (a) $168(10.5) + 94{\cdot}5(7) = 262{\cdot}5\bar{x}$ $\bar{x} = 9{\cdot}24$ — M1 A1 A1
   (b) $168(4) + 94{\cdot}5(11) = 262{\cdot}5\bar{y}$ $\bar{y} = 6{\cdot}52$ — M1 A1 A1
   (ii) $\tan \alpha = (21 - 9{\cdot}24)/6{\cdot}52 = 1{\cdot}804$ $\alpha = 61{\cdot}0^0$ — M1 A1 — 8
5. (i) Momentum : $4mu - 5mu = mv_P + mv_Q$ $v_P + v_Q = -u$ — B1
   Elasticity : $(v_p - v_Q)/(-5u - 4u) = e$ $v_P - v_Q = -9eu$ — M1 A1
   Subt : $2v_Q = -u + 9eu$ $v_Q = \frac{1}{2}(9e - 1)u$ — M1 A1
   (ii) $v_Q > 0$, so $9e > 1$ $e > \frac{1}{9}$ — M1 A1
   (iii) $v_P = -\frac{1}{2}(9e + 1)u$ After hitting wall, speed of $Q < \frac{1}{2}(9e - 1)u$ — M1 A1
   which is clearly less than $|v_P|$, so there is no further collision — A1 — 10
6. (i) $7 = \frac{1}{2}gt^2$ $t^2 = 14 \div 9{\cdot}8$ $t = 1{\cdot}195$ — M1 A1
   In 1·195 s, stone travels $4 \times 1{\cdot}195 = 4{\cdot}78$ m — A1
   (ii) When $x = 2{\cdot}39$, $t = 0{\cdot}598$ $y = 7 - \frac{1}{2}gt^2 = 5{\cdot}25$ m — M1 A1 A1
   (iii) $mgh = 1{\cdot}5 \times 9{\cdot}8 \times 7 = 102{\cdot}9$ J — M1 A1
   (iv) $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$ $v = \sqrt{(14g + 16)} = 12{\cdot}4 \text{ ms}^{-1}$ — M1 A1
   (v) Modelled stone as particle, ignored air resistance, etc. — B1 B1 — 12
7. (i) For Q : $T = mg$ For P : $T = m(0{\cdot}2)\omega^2$ — B1 M1
   $\omega^2 = g/0{\cdot}2 = 49$ $\omega = 7$ No. of r.p.m. $= {}^7/_{2\pi} \times 60 = 66{\cdot}8$ — A1 A1
   (ii) For Q : $T \sin 45^0 = m(0{\cdot}2)\omega_1^2$, $T \cos 45^0 = mg$ — M1 A1 A1
   $\tan 45^0 = 0{\cdot}2\,\omega_1^2 / g$ $\omega_1^2 = 49 \tan 45^0 = 49$ — M1 A1
   For P : $T = m(0{\cdot}2)\omega^2$ But $T = mg\sqrt{2}$ so $\omega^2 = 49\sqrt{2}$ — M1 A1
   $\omega^2 : \omega_1^2 = \sqrt{2} : 1$ $\omega : \omega_1 = 2^{\frac{1}{4}} : 1$ — A1 — 12